

Masterclass Session Script

The Maths of Voting	Slides/ Worksheets
<p>Welcome to today's Masterclass. In this session, we'll be exploring the maths behind voting systems, how winners are calculated in different electoral systems and to what extent these systems are fair.</p> <p>Does anyone have any suggestions for the best colour (/whatever question you might change this to)?</p> <p>By the end of this session, you'll know how to run an election to answer this definitively!</p> <p>That said, keep in mind that there are many different voting systems and we'll only be looking at a few of them today.</p>	Slide 1
<p>I have some examples of things we vote for on the slide and as you can see elections are a fundamental part for democracy and political life, but voting is part of our everyday lives in ways that we might not always recognise. People vote to make important decisions for their country, like electing who to run the country or, for example, whether or not to leave the European Union. Likewise, people might vote for their favourite contestants in a show like I'm a Celebrity or Strictly.</p>	Slide 2
<p>So why do we vote?</p> <p>Well, voting is important – in a democratic society, citizens must be allowed to have their opinions taken into consideration on important topics.</p> <p>Maths and voting are connected, in that maths can be used to design a voting system and judge its fairness, and we'll look today at how maths is involved in the election process.</p>	Slide 3
<p>Now, sometimes elections don't exactly go the way we expected.</p> <p>In the 2016 US election, for example, Donald Trump was elected President but with only 46% of the vote, whereas Hilary Clinton had 48%. This is because in the US, to win the election you need to have the most votes in enough states – in 2016, Trump achieved this without actually having the most votes overall, called the popular vote.</p> <p>In the 2017 UK General election, Theresa May, the Prime Minister at the time, wanted a stronger majority for her Conservative Party, but ended up with what's called a hung Parliament, where no party has over half of the seats in Parliament. This would make it hard for her to make things happen in Parliament, as she could have a lot of people voting against her. To gain the last few seats needed, she formed a coalition with the Democratic Unionist Party, giving power to a party with only 10 seats.</p>	Slide 4

<p>There are also many countries where even a clear victory is not exactly straightforward. Belarus has only ever had one President, being Alexander Lukashenko who began his Presidency in 1994. In 2020, he won the election with 80% of the vote, which raised a lot of eyebrows as to the legitimacy of the election.</p> <p>Suffice to say, elections can often lead to unexpected or unusual results, depending on the system in place.</p>	
<p>Before we look at some electoral methods, I want you to spend 1 or 2 minutes speaking to the person next to you about what makes a system fair in your opinion.</p> <p><i>Give a few minutes for discussion</i></p> <ul style="list-style-type: none"> • What did you talk about? 	Slide 5
<p>One of the main ways people have thought about and evaluated whether a system is fair is through Arrow's theorem. The theorem was thought up by Kenneth Arrow, who you can see in the slide, in the 1950s and it was part of the reason that he was awarded the Nobel Prize of Economics in 1972.</p> <p>According to Arrow, for a system to be fair it must meet four criteria:</p> <ul style="list-style-type: none"> • All votes must be equal • If everyone prefers one candidate, then that candidate must win • There should be one clear winner • Whether A or B is preferred should not depend on other candidates 	Slide 6
<p>Let's look at how the theorem works using a practical example. Let's say we have three siblings, Kendall, Siobhan and Connor, and that they are trying to decide what flavour of ice-cream to buy. They can choose between chocolate, vanilla and strawberry. On the slide, you can see how they ranked each flavour from favourite to least favourite.</p> <p>For the system to be fair, according to Arrow, everyone's vote needs to be equal – so Kendall's vote, for example, can't have more weight than any of the others. In this case, everyone prefers chocolate so chocolate must win. Elections are expensive and need a lot of planning, so we want a system that will give us a clear winner which is chocolate.</p> <p>The last criteria is a bit more difficult, but it means that if after the siblings had written their preferences (and chocolate had won), if I added another flavour, say mint choc chip, the winners could only be chocolate or mint choc chip.</p> <p>We might also consider other elements that are needed for a fair system, like the right to an anonymous vote or a winner that the majority want.</p>	Slide 7

<p>We'll be returning to Arrow's theorem throughout the Masterclass, but for now keep these ideas of fairness in mind.</p>	
<p>Today we're going to analyse how different electoral systems work, how they impact the winner, and whether we think they are fair. We're going to be focusing on three voting methods: First Past the Post, Alternative Vote and Borda count. These methods all have uses today – First Past the Post is the system we use in the UK for national elections. Alternative Vote is used in Australia and New Zealand; we voted in 2011 whether to change to AV too, but over 60% of people voted against it. Finally, Borda is a system that doesn't see as much use in national elections, but is used in some countries like Nauru in Oceania. Borda is more commonly used in scoring competitions, as we will see later.</p>	Slide 8
<p>Let's start by using First Past the Post (FPTP). This is a very simple system: everyone votes for their favourite candidate, and the candidate with most votes wins.</p> <p><i>If you would like a quick demonstration:</i> We're going to give this a try ourselves. Look at the pizza varieties on the right and raise your hand when I call out your favourite. <i>(Call out the pizza options one by one and tally up the votes, then announce the winner)</i></p>	Slide 9
<p>To look at how FPTP works, we're going to use an everyday example of voting. We have a family of 12 – from A to L – that is deciding on what type of pizza to order. You can see their choices on the slide and what each symbol represents. So, A wants a four Cheese pizza, B picks Veggie, C margherita, and so on. <i>(Get students to work out which pizza wins – ham and pineapple)</i></p> <ul style="list-style-type: none"> • So, looking at everyone's choices, which pizza wins? <p><i>Click for next slide</i> Ham and pineapple has three votes, which is the most out of any topping and, therefore, wins.</p> <p><i>Click for next slide</i></p> <ul style="list-style-type: none"> • But, looking at everyone's choices, do you think this is a good or fair outcome? Why? <p><i>Go back to slide with all votes if needed. If you ran a mock election earlier also, ask students to think about whether that outcome was fair and how they felt about the results.</i></p>	Slide 10 - 12
<p>Looking at everyone's preferences, you can see that most people voted for other toppings and did not want ham and pineapple. In fact, only 25% of the family voted for ham and pineapple and the rest effectively voted against it.</p> <ul style="list-style-type: none"> • Is this a fair outcome? What if someone can't eat ham, or is allergic to pineapple? 	Slide 13

<p>Let's look at what happens when we use FPTP at the national level. For general elections, the country is divided into over 600 constituencies. You can see in this map on the right how these are divided and also how they voted in the last election.</p> <p>Each constituency uses FPTP to elect a local MP – so effectively, over 600 mini elections are happening across the country – and the party with most seats, that is, most constituencies becomes the ruling party. As we've already talked about with the 2017 General Election example, sometimes this doesn't work out and parties need to form coalitions but this is rare.</p>	Slide 14
<p>In the graph, you can see the percentage of votes (in green) compared to the percentage of seats obtained (in blue) by the five parties with most votes in the 2024 election.</p> <ul style="list-style-type: none"> • What do you make of it? Is this what you would expect? <p>Something that might seem strange to you is that even though Liberal Democrats and Reform had a similar number of votes, Lib Dems have far more seats. This is because Lib Dem voters were typically concentrated into constituencies with a large Lib Dem voting population, while Reform voters were sort of diffused across the country so couldn't get the most votes in many constituencies.</p>	Slide 15
<p>Strategically changing boundaries to help win an election is called "gerrymandering".</p> <p>The term originated in the 19th century when governor of Massachusetts in the US, Governor Eldridge Gerry, signed a bill which changed the boundaries of his constituency to improve his chances of winning the upcoming elections.</p> <p><i>Follow the hyperlink on the slides, and scroll through the highly visual and dynamic article.</i></p> <p>This is a real-life example of how changing the boundaries in North Caroline to diffuse Democrat voters can turn a predominantly Democrat area into a Republican one – and change who the overall winner in that area is.</p>	Slide 16
<p>So, we've seen how gerrymandering can tip the scales in favour of one party, but can you gerrymander?</p> <p>First:</p> <ul style="list-style-type: none"> • If there are 6 constituencies, how many does Magenta need to win to have a majority overall? <p>Magenta needs at least 4 constituencies.</p> <ul style="list-style-type: none"> • What happens if there is a tie? <p>If there is a tie, then nobody wins the constituency.</p>	Slide 17 - 18 Worksheet 1

<p>Once most students look to have finished the first two maps, click to the next slide:</p> <ul style="list-style-type: none"> • Has anyone managed to do this for maps 1 and 2, following all the rules? • What about map 3, did anyone manage to give Magenta a majority? Why not? <p>You need at least 4 Magenta voters to make a Magenta winning constituency and Magenta needs to overall win four constituencies to have a majority. This means that Magenta needs $4 \times 4 = 16$ Magenta voters overall but there are only 14 in this area, so mathematically it can't have a majority.</p>	
<p>To avoid 'wasting' their vote, sometimes people don't vote for the party that they prefer or aligns the most with their views, they don't go for their first choice because it's highly unlikely that it will win. Instead, they vote strategically, so they vote for a party they might like less but that has a higher chance of succeeding. This is called tactical voting.</p> <p>In our pizza example, this tactical voting might look like H and J who prefer BBQ and anchovy pizzas to change their vote to Four Cheese, for example, to avoid a ham and pineapple win.</p> <p><i>Click for next slide</i></p> <p>And, in this case, with H and J's votes, Four Cheese would be the winner.</p> <p>This is something that happens a lot with FPTP on the national level – if someone lives in a constituency that always votes for a party or candidate that they really don't like, they might vote for the candidate that's most likely to beat them, rather than their actual favourite.</p>	Slide 19 – 20
<p>With all of this in mind and Arrow's criteria, to what extent do you think FPTP is fair?</p> <p><i>Allow a few minutes for discussion; once conversation is over, go to comfort break</i></p>	Slide 21
<p>Comfort break.</p>	
<p>We've looked at what happens when you ask for people's favourite or first choice pizza, and now we're going to analyse what happens when you ask for more information. So, instead of just writing down 'cheese' or 'pepperoni', every family member ranks the options from 1 to 4. (<i>Note: there are only now four options to make it easier for students.</i>)</p> <ul style="list-style-type: none"> • Which pizza would win here if we were still voting with First Past the Post? <p>Ham and Pineapple would be the winner here still.</p>	Slide 22

<p>In the Alternative Vote or AV, the winner needs to have the majority of the total votes, not just the most votes like in FPTP.</p> <p>As we briefly discussed before, AV is used in countries like Australia. We actually had a referendum in 2011 to change FPTP to AV, but 67.9% of voters against it.</p> <p>If you want a quick demonstration: Now we're going to quickly demonstrate how Alternative Vote could work in terms of a simple vote with two candidates – red or blue. I'm going to split the classroom in half and...</p> <ul style="list-style-type: none"> • (If there is an even number of students, split the class in half) and now if I move (choose a student on one side at random to move) to the other side, then (that side) wins! • (If there is an odd number of students, split the class in half, leaving one student at random in the middle) now you can choose which side you prefer (student chooses a side) and that side wins! <p>As we can see, in a simple Alternative Vote system, all we need is to cross that 50% threshold to determine a clear winner.</p>	<p>Slide 23</p>
<p>Let's look at an example of the preferences of five people on what flavour of ice-cream they want to understand how AV works.</p> <p>In AV, you start by calculating what the majority of votes is.</p> <p>Can anyone tell me what the majority is?</p> <p>A majority is more than half, so, in this case, half of five is 2.5 and, since you can't have half a vote, the majority needed is 3 votes.</p> <p><i>Click for next slide</i> Looking only at the first preferences, does any ice cream have a majority?</p> <p>Strawberry and chocolate both have 2 votes but neither has a majority, so we eliminate the flavour with the least number of votes – which is vanilla – and re-distribute its votes.</p> <p><i>Click to next slide</i> Strawberry moves to C's first preference and, having now 3 votes, strawberry reaches majority and is the winner.</p>	<p>Slides 24 - 27</p>
<p>In your worksheet, you have the family's preferences and a table to work out the majority and which topping wins.</p> <p>Hand out Worksheet 2</p> <ul style="list-style-type: none"> • Which pizza is the winner? <p><i>Click to next slide once students are mostly done</i> Cheese pizza wins this time!</p> <ul style="list-style-type: none"> • How does AV compare to First Past the Post? 	<p>Slides 28 – 29 Worksheet 2</p>

<p>The Borda count or method was designed in the 18th Century, and it transforms the table of preferences into a massive calculation, so each ranking gets points attributed to it.</p> <p>So how do you know what points to give each ranking in Borda? You do it by, first, determining how many candidates are running, that is your P. Then, you do P-1, P-2, P-3 and so forth to determine each ranking's points. The last preference is given 0 points. If we have five candidates, the rankings will be as shown in the slide.</p> <ul style="list-style-type: none"> • Can you think of a real-life example that uses a similar method? (If they're struggling, maybe mention competitions or championships.) 	Slide 30
<p>In Formula 1, drivers get between 25 and 1 points if they finish within the first 10 places. Similarly, in Eurovision, each country awards points to their favourite songs. These are adapted versions but are still built on Borda count, fundamentally.</p>	Slide 31
<p>Hand out Worksheet 3. In the worksheet, you have the family's preferences and a table to work out each pizza's points.</p> <ul style="list-style-type: none"> • Which pizza wins using Borda? 	Slide 32 Worksheet 3
<p>That's right, there is a tie between Cheese and Pepperoni, so we don't have a 'proper' winner.</p> <ul style="list-style-type: none"> • Is this a good outcome? Why/why not? • What are some ways we could break the tie? <p>In an actual election, the most likely outcome would be running a new election with only cheese and pepperoni to decide the winner, but this is costly and takes time.</p>	Slide 33
<ul style="list-style-type: none"> • Did any of these electoral systems satisfy all of Arrow's conditions? 	Slide 34
<p>Today, we have looked at First Past the Post, Alternative Vote, and Borda, and each system has produced a different result.</p> <p>A key problem with FPTP is that very often, people do not actually vote with their real preferences, instead voting tactically. This means that there is practically no chance of finding a universal preference that might exist, breaking rule 2.</p> <p>In Alternative Vote, if we were to change the candidates up for election, then the outcome may change for some of the candidates that haven't changed. This breaks rule 4.</p>	Slide 35

<p>In our example, Borda produced a tie, and this can happen in real life too. As such, Borda breaks rule 3.</p> <p>Overall, none of our systems follow all of the criteria for a 'fair' system.</p>	
<p>Actually, Arrow's theorem is known as the impossibility theorem because no electoral system can meet all the criteria if there are three or more candidates running. This means that none of the systems we have seen already, and none of the others we haven't, can be completely fair according to Arrow – they are all imperfect in some way.</p>	Slide 36
<p>So, for the last part of today's Masterclass, we're going to see if you can come up with a better, fairer system.</p> <p>You have a worksheet with all of this information if you get lost at any point or want some pointers.</p> <p>New countries have been created and it is your job, in groups, to decide on which voting system will be used in your country's elections. To do so, you will create, adapt or pick randomly a system and use it to run a mock election. You have 5 minutes to decide on a system – starting now!</p> <p><i>Once students have selected a system (especially if they have chosen a random system) click through slides 42 and 43 to reveal the random systems. Hand out Random systems instructions.</i></p>	<p>Slide 37 – 39</p> <p>Worksheet 4 Random systems instructions (if needed)</p>
<p>Now that you have a system in mind, you will run a mock election for the other groups. They will vote for their favourite primary colour: red, yellow, or blue or, depending on the electoral method you're using, rank them in order of preference.</p> <p>Use the Ballot Handout provided to write/give clear instructions to the other groups about how they should fill that ballot, like whether they need to rank colours, or pick just one. You can use the ballot as is, or you can use it as a template to make your own.</p> <p><i>Hand out Ballot Handout, and give students a few minutes to get everything organised.</i></p> <p>Now that you have an electoral system, ballots and instructions, it's time to give these to the other groups. Assign someone from your group to collect the ballots once everyone has filled theirs and, together, count them and determine which colour is the winner, using your system. You have 10 minutes to do this!</p>	Slide 40 Ballot handout
<p>Now that every group has run an election and determined the winner, you will discuss what system you used, which colour won and whether you think this is the system that should be adopted going forward. This time, you have just 5 minutes so keep your descriptions brief!</p>	Slide 41

<ul style="list-style-type: none"> • Coming together as the whole group now, can someone from each country tell me what system they have picked and why? 	Slide 42
<p>In today's Masterclass, we've looked at the maths behind different voting systems and considered whether they are fair. We also experimented with creating different systems and running small elections with them as well as considering the pros and cons of adopting electoral systems in real life.</p> <p>Next time you hear someone talking about FPTP you will be able to explain to them how it works and how it compares to other electoral systems.</p> <p>Before we go, are there any questions or thoughts you'd like to share?</p>	Slide 43