

Thank you very much for leading this Masterclass. We hope that you enjoy working with this material as much as we enjoyed putting the activities together. We do appreciate all the effort that our volunteers put into bringing inspirational Mathematics Masterclasses to students around the country. Don't forget that we'd love to know your thoughts on the Masterclass – more detail at the end of this section.

#### Inspiration for this topic:

Models are useful tools that enable us to describe the real world and predict many things like the weather, population dynamics, the spreading of disease, the impact of an earthquake... In this session, the students are introduced to cellular automata: mathematical models composed of grids of cells that evolve over time according to simple rules based on the states of neighbouring cells.

Students will use cellular automata to model forest fires on paper grids, first with a given set of parameters; they can then devise their own set of logical rules to change the model, making it their own!

Length: 2.5 hours.

#### Overview of Activities:

1. Worksheet 1: Spreading Dye (to get a handle on the idea of cellular automata)
2. Worksheet 2: Forest Fires (first model of spreading fire among discrete trees)
3. Worksheet 3: Your Adaptations (adding new conditions to the forest fire)

#### Specific resources needed (enough for the number of students attending):

- One copy of Worksheet 1 and one copy of Worksheet 3 per student (each 2 sides)
- One copy of Worksheet 2 per group of up to 4 students (8 sides)
- Dice: 3x 6-sided and 1x 10-sided (or equivalent) per group of up to 4 students;
- Additional dice for students to borrow for worksheet 3 (if possible, have a selection of different shapes, e.g. 20-sided, 12-sided, 10-sided, 8-sided, 6-sided and 4-sided)
- One set of coloured pencils per group
- Students will also need paper and a pen/pencil.

If you don't have dice, students can also use:

- A dice simulation app or website
- The random number generator on a calculator (this is not truly random, but it will work OK for our purposes)
- Pieces of paper with numbers on, folded up and chosen at random out of a cup (remember to replace the paper after each turn)
- The correct number of playing cards, shuffled thoroughly after each turn
- Different coloured objects which are the same size, shape and feel, picked at random out of a bag and replaced after each turn

#### Support resources:

- PowerPoint slides
- Helper notes
- Session script
- Additional information
- Worksheet solutions

### Things to prepare in advance:

- Print worksheets (can be double-sided) and gather the complete list of resources as detailed above. **Print these from PDF or the Desktop version of Word, not online/browser.**
- Print helper notes, session script, additional information if needed. **Print these from PDF or the Desktop version of Word also.**
- Gather general Masterclass resources

### Ask the Ri

Don't forget to collect any questions which arise, and email them to the Masterclass team at the Royal Institution: [masterclasses@ri.ac.uk](mailto:masterclasses@ri.ac.uk)

### Feedback

We would very much welcome your feedback on this session. If you have time, please collect feedback from the students at the end of the Masterclass and send it through to us. We would also appreciate feedback on how you have used the session, what you think worked well and what improvements would be useful.

## Time plan of Masterclass:

Slides & Time	Title & Overview	Activity (see script for further details)
<p><b>Slide 1</b></p> <p>5 minutes</p> <p>(5)</p>	<p><b>Mathematical Modelling</b></p> <p>Introduction</p>	<p><b>Settling question:</b> have students think about the factors that might impact forest fires. Once everyone is settled tell them to keep those ideas in the back of their head, then move on to the Masterclass.</p> <p><b>If you need a longer settling activity</b>, students can make a start on <b>Page 2 of Worksheet 1: Spreading Dye</b>. Instructions are on the Worksheet already. Tell students not to turn over the page.</p> <p>Introduce students to modelling as a concept and its uses.</p> <ul style="list-style-type: none"> <li>• Models are useful tools that help describe and predict many things of the world around us: the weather, the spread of disease, the stock market, etc.</li> <li>• A mathematical model is a set of equations/algorithms used to predict real-world behaviour.</li> <li>• Introduction to discrete vs continuous time; example of counting a population of rabbits in discrete time i.e., in steps.</li> </ul> <p>Today's objective: use mathematical modelling to predict the spread of forest fires.</p>
<p><b>Slide 2</b></p> <p>5 minutes</p> <p>(10)</p>	<p><b>Cellular Automata</b></p> <p>General characteristics</p>	<p>Characteristics of cellular automata appear in bullet points with each click.</p> <p>A table showing all eight neighbours of a given cell is provided to illustrate. The green dot in the middle is 'active', while its neighbours are considered inactive. A cell may become active from inactive if, say, it has a direct neighbour that is active.</p>
<p><b>Slides 3</b></p> <p>5 minutes</p> <p>(15)</p>	<p><b>Spreading dye</b></p> <p>Working principles</p> <p><b>Worksheet 1</b></p>	<p><b>Hand out Worksheet 1: Spreading Dye, if not already handed out during settling. Students to wait for instructions. NB if you have used page 2 of this worksheet for a starter activity, just ask the students to turn over.</b></p> <p>Text appears with consecutive clicks.</p> <p>Emphasise the fact that for each time-step, we want to know which squares are about to be dyed, not the ones that already are.</p>
<p><b>Slides 4-6</b></p> <p>25 minutes</p> <p>(40)</p>	<p><b>Spreading dye</b></p> <p>Example</p> <p><b>Worksheet 1</b></p>	<p>Example to start with <b>Worksheet 1: Spreading Dye</b>.</p> <p>Work through time steps 0, 1, and 2 in the example shown.</p> <p>Let students work through Worksheet 1, which is another example of the spreading dye you just did together. Students who finish quickly and/or didn't get chance earlier can try the Game of Life activity on Page 2.</p>
<p><b>Slide 7</b></p> <p>5 minutes</p> <p>(45)</p>	<p><b>Forest fires</b></p> <p>Principles of modelling forest fires</p>	<p>Using cellular automata to predict the spread of forest fires in a discrete (non-continuous) way.</p> <p>Modelling is important as we want to know how to stop forest fires.</p> <p>Ask students for parameters of spreading fires. Three bullet points are animated.</p>

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<p><b>Slide 8</b></p> <p>10 minutes</p> <p>(55)</p>	<p><b>Introducing chance</b></p>	<p>In the forest fire model, chance is introduced as probability.</p> <p>Each sentence appears in succession. A grid is given by example to show how to use dice to assess whether trees are burning depending on probability:</p> <ul style="list-style-type: none"> <li>• Tree D has one burning neighbour: roll dice once.</li> <li>• Tree E has two burning neighbours: roll dice twice.</li> <li>• Tree F has three burning neighbours: roll dice three times.</li> </ul> <p><b>Example</b> – show on board or visualiser. Get a volunteer to be your ‘dice roller’. If you can, use different colours or patterns to fill the squares in each time-step.</p> <ul style="list-style-type: none"> <li>- Start with a grid – 5x5 or bigger. Colour in the middle square – this is your burning tree in time-step 0.</li> <li>- Using a new colour, put a dot in each of the ‘trees’ (squares) immediately surrounding your coloured square to show that these have a possibility of burning. They will burn with probability 1/3 in this example – get the students to shout out two numbers to look for on the six-sided die, if those numbers are rolled then the tree you are checking will catch fire.</li> <li>- Choose a starting point (one of the dotted squares) to check; does the tree catch fire? Get your dice roller to roll the die and see. If yes, colour it in in the colour/pattern for time-step 1; if not, move to the next dotted tree. For the first time step, you only need to check all the dotted trees once each.</li> <li>- When you get your first burning tree: when checking the next tree, ask how many times you need to roll the dice. Students to shout out the answer - once. Remind them that the tree you have just coloured in is not on fire yet, so fire cannot spread from there; it will only be on fire at the end of the time-step. To make it easy to do, you are using a different colour/pattern for each time-step, and you need to ignore any trees with the colour/patterns that you are currently using.</li> <li>- Once you have got back to your starting point, tell the students that this is the end of time-step 1, and any trees you have just coloured in are now on fire. There is a small chance that no new trees will be on fire: in this case the next time-step will follow the same process as above.</li> <li>- Switch to a new colour and put a dot in any trees which can now catch fire; remind the students that any trees which already had a dot can still catch fire.</li> <li>- Choose a tree to check which has more than one burning neighbour. Get the dice roller to roll once; does it catch fire? If not, ask the students if you need to roll again (yes). If it catches fire first time, talk about what would have happened if it hadn’t (i.e., needing to roll again for the other burning neighbour tree). Remember that the number of times you need to check equals the number of burning neighbours.</li> </ul> <p>Once you've illustrated the ‘multiple burning neighbours’ scenario, you can move on – you don’t need to finish the time-step.</p> <p>Discuss what weather conditions would affect the model.</p>

<b>Slides &amp; Time</b>	<b>Title &amp; Overview</b>	<b>Activity (see script for further details)</b>
<b>Break</b> 15 minutes (70)		Comfort break
<b>Slides 9-10</b> 30 – 40 minutes (110)	<b>Conditions</b> Probabilities used in <b>Worksheet 2</b>	<p><b>Hand out Worksheet 2: Forest Fires.</b></p> <p>Discuss conditions affecting whether a tree might catch fire. This changes the probability from 1/3. Normal, dry, drought, wet weather/flood appear one after another.  <b>You can change the probabilities given here depending on the available dice.</b></p> <p><i>You can choose to put break time during the worksheet time. They can carry on with their worksheets during the break if they want to. After the break and Worksheet 1 you should have ~40 minutes left.</i></p> <p>Ensure all students have <b>Worksheet 2: Forest Fires</b> – there is one per group of (up to) 4, and they will each have a double-sided page with a different probability to check. Each group of 4 will also need coloured pencils and three six-sided dice and one 10-sided die. If you don't have any dice, get them to use folded bits of paper, or a phone app if some of them have phones. If you don't have 10-sided dice, adjust the probabilities on the worksheet and slides (remember to test!).</p> <p>If they finish looking at their condition once, it is important to do it again to see if the pattern is similar – because chance is involved, it probably won't be! Encourage them to either repeat the same condition again or swap with another person in their group.</p> <p>Discuss outcomes.</p>

Slides & Time	Title & Overview	Activity (see script for further details)
<p><b>Slide 11</b></p> <p>25 minutes</p> <p>(135)</p>	<p><b>Fire modelling</b> Adding conditions <b>Worksheet 3</b></p>	<p><b>Hand out Worksheet 3: Your Adaptations.</b></p> <p>Points appear with animations.</p> <p>Students discuss the outcomes of <b>Worksheet 2: Forest Fires</b> in small groups. Lead a short discussion about what the students found and the patterns they noticed – can also ask for hands up if different things happened, e.g.:</p> <ul style="list-style-type: none"> <li>- Did any of them fill in their grids within a certain number of time-steps?</li> <li>- Did any have a single unburnt tree for several time-steps?</li> <li>- Did any of them have fires which grew in particular directions?</li> <li>- Did any not completely fill their grids?</li> </ul> <p>What conditions were each of these things for? Talk/ask about whether they will all have got the same/similar patterns, given chance was involved. They may have some really interesting thoughts about what they have noticed.</p> <p>Discuss the other factors which could be incorporated. Give thinking/discussion time. Write answers on the board as they arise e.g., “wind speed”, “wind direction”; pupils can use these ideas later.</p> <p>Students work on <b>Worksheet 3: Your Adaptations.</b> Encourage them to be creative: there is no right answer, but the conditions must make sense!</p> <p>Put out a selection of dice; if working with an older group than Year 9 or have lots of extra time in your scheduled workshop, you could also have different paper (e.g., hexagonal or triangular grids, that provide a different number of neighbours to each burning tree).</p> <p>Go round and talk to the students; check what they are including, and how they are including it. Make sure this makes sense (e.g., wind direction would affect the probabilities of trees relative to the burning tree, not give a different probability for one half of the grid). Check all understand and are happy working on their models.</p> <p>Stop 10 minutes before the end (longer with a very large group) and go round the room asking the students what they added into their model, in the opposite order to when you asked for feedback on worksheet 2. Tell the students to keep it brief – e.g., “wind direction” rather than explaining how, though for some ideas a bit of explanation might be required. If you’re tight on time you can also ask for hands up to show who else used a common idea.</p>
<p><b>Slide 12</b></p> <p>5-10 minutes</p> <p>(145)</p>	<p>Further activities</p> <p>Feedback, tidy up, questions time</p> <p>Ask the Ri</p>	<p>Introduce the extension material shown.</p> <p>Don’t forget to collect any questions which arise, and email them to us: <a href="mailto:masterclasses@ri.ac.uk">masterclasses@ri.ac.uk</a></p> <p>We will send you answers as soon as possible. Then these can be reported back to the students at their next Masterclass session. In this way you cannot be “caught out” by a question. It also demonstrates the point that not everything in maths is known, but some questions need time and research to find answers sometimes, and sometimes the answer has not been found by anyone yet, of course! Maybe our Masterclass students will be the ones who solve the problem when they are older?</p>